Comment on “Isaac Newton vs. Red Light Cameras: Derivation of the Yellow Light Interval Equation” by Brian Ceccarelli

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Introduction

Ceccarelli (2010) argues that the equation for calculating the appropriate yellow interval for a signalized intersection, as specified in the Institute of Transportation Engineers (ITE, 2009) Traffic Engineering Handbook, is unsafe because it yields yellow light intervals that are too short for drivers to stop before the intersection without extreme braking effort. The latter is correct. Recommended yellow light intervals are shorter than times required for reasonable drivers to come to a stop at the stop line, but the conclusion that this makes the intersection unsafe is incorrect. Ceccarelli also is correct that signal timing must be set in accordance with Sir Isaac Newton’s laws of motion, but his application of those laws is wrong.

To understand how it is wrong, it is necessary first to understand a traffic engineer’s task in timing intersection signals. Basically, the engineer has to ensure the transition of flow from one direction to the other occurs safely. This depends on two issues: how much distance reasonable drivers require to stop safely at the intersection and how much time is needed for drivers who are closer than that to the intersection to clear. Contrary to Ceccarelli’s (2010) argument, these latter drivers — the ones who cannot stop because they are too close — determine the length of the yellow interval because they are in danger of crashing if the cross traffic receives a green light too soon. However, the yellow interval still must be based on the distance a reasonable driver requires to stop. The best way to understand this is by an example.

Suppose traffic is approaching an intersection with a speed limit of 35 miles per hour (mi/h), or 51.3 feet per second (ft/sec). There is no grade (the road is level), and the 85th percentile speed is less than the speed limit. So that the speed limit provides a reasonable upper bound to accommodate drivers’ speeds.

The first step to establishing an appropriate yellow interval is to calculate the stopping distance needed by drivers proceeding on green to perceive that they can safely stop without emergency braking or extreme maneuvers. This is given by the equation:

\[ S = v_o \left( t_p + \frac{v_o}{2(a_b + 32.2 \ g)} \right), \]

where: 
- \( v_o \) = “speed limit, unless a speed study determines that the 85th percentile speed is faster or intersection geometrics compel vehicles to traverse the intersection slower,” ft/sec;
- \( t_p \) = driver perception/reaction time, sec;
- \( a_b \) = vehicle deceleration rate, ft/sec\(^2\);
- \( g \) = grade of approach, %.
If driver perception/reaction time is taken to be 1.5 sec with a deceleration rate of 11.2 ft/sec² — numbers widely accepted as reasonable for alert drivers (e.g., _Intelligent Transportation Systems and Signals Unit Design Manual_ (North Carolina Department of Transportation, 2004)) — then:

\[ S = 51.3 \times (1.5 + 51.3/2(11.2 + 0)) = 195 \text{ ft} \]

for that approach.

This establishes that a reasonable driver traveling at the speed limit needs 195 ft to stop comfortably when the light changes from green. Drivers traveling the speed limit but at least 195 ft from the intersection when the green light changes can be expected to come to a stop. Drivers closer than 195 ft are expected to continue through because they are too close to stop comfortably (unless they are traveling more slowly). These are the drivers in danger of colliding with cross traffic if they stop in the middle of the intersection after the light turns red and the cross traffic receives a green. The yellow interval must be long enough for them to enter on yellow without braking.

To estimate the yellow light duration, the traffic engineer calculates the time it would take drivers traveling the speed limit to enter the intersection. That time is given by the equation:

\[ \text{ITE yellow interval} = \frac{S}{v_0} = t_p + \frac{v_0}{2(a_b + 32.2 \text{ g})}. \]

That is, the yellow light interval is given by the distance needed for a reasonable driver to stop (which also is the longest distance expected for a driver to decide to proceed through after the green light ends) divided by the speed of the driver. Assuming the speed to be 35 mi/h (51.3 ft/sec), the necessary yellow interval for this example equals 3.8 sec. That is, given these conditions, all drivers would be expected to be able to stop before the intersection or be able to enter the intersection on yellow, before the light turns red.

[Note that this is the minimum recommended yellow interval to allow drivers to enter the intersection legally, before the light turns red. To ensure traffic that does not stop has time to clear the intersection before cross traffic begins, engineers may either lengthen the yellow interval or add a period when the signal is red for all traffic.]

**Ceccarelli’s Paradox, Revisited**

Ceccarelli (2010) points out that this yellow interval is shorter than the estimated time for drivers traveling at the speed limit to come to a reasonable stop. Specifically, the time needed for drivers to come to a comfortable stop is given by the equation:

\[ \text{Stopping time} = T = t_p + \frac{v_0}{(a_b + 32.2 \text{ g})}. \]

For our example, \(T\) is 6.1 sec, or 2.3 sec longer than the required yellow interval. Why is this still safe? What explains this seeming paradox? This is safe because vehicles have enough distance to stop
safely and comfortably before the intersection; it takes longer than the yellow interval to stop because drivers are slowing down and covering the distance to the stop line at a slower speed than drivers nearer the intersection who decide to proceed through. Again, an example may help clarify this point.

Assume a vehicle traveling at 35 mi/h (51.3 ft/sec) approaches the intersection when the light turns yellow. This vehicle is 200 ft away from the intersection stop line and the driver starts to brake at a deceleration rate of 11.2 ft/sec². At the end of the ITE yellow interval (3.8 sec), this vehicle’s traveling speed is 25.7 ft/sec, and the traveled distance is 165 ft from the start of yellow light. In other words, at the end of the ITE yellow interval and the start of the red interval, this vehicle is still 35 ft (200-165 ft) away from the intersection stop line and still is decelerating. In another 2.3 sec after the start of the red interval, the vehicle comes to a complete stop, still 5 ft away from the stop line. There is no need to add the 2.3 sec to the ITE yellow interval (3.8 sec), because this vehicle can stop completely and safely prior to the stop line during the red interval. Other, similar vehicles closer to the intersection can pass through before the light turns red.

In short, there is no paradox. The ITE yellow interval is designed to protect vehicles too close to the intersection to stop comfortably when the green interval ends. The yellow interval does not determine the stopping time of those who choose to stop, because their deceleration and slower average speed as they stop means they have longer than the yellow interval to achieve their stop.

References

